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SYNTHESIS OF ADDITIVE AMBIENT SEISMIC NOISE WITH A GAUSSIAN MARKOV MODEL

19 May 1967

Prepared For

AIR FORCE TECHNICAL APPLICATIONS CENTER
Washington, D. C.

By

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TELEDYNE, INC.

Under

Project VELA UNIFORM

Sponsored By

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SYNTHESIS OF ADDITIVE AMBIENT SEISMIC NOISE WITH A GAUSSIAN MARKOV MODEL

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ABSTRACT

The ambient seismic noise is modeled by a single Gaussian population from which independent realizations or states are taken as input to tuned filters with spectral peaks matched to those observed in noise samples, for example at .2 cps and 2. cps. For each spectral noise peak, the realization on channel i + 1 is equal to a constant times the realization on channel i plus another constant times a new realization on channel i + 1. The constants defining the Markov process can be used to theoretically derive the associated power spectral matrix of the noise model. The model can be extended to dispersive systems by using a set of constants and time lags to relate the noise on channel i to that on channel i + 1.

A vertical array signal model is also given. The purpose is to efficiently generate noise and/or signals at prescribed S/N ratios. The noise covariance structure is close to that observed naturally and is known exactly for the noise model realizations. Thus the spectral covariance of the noise is given exactly subject only to roundoff error, and conditions of stationarity and equilibrium are satisfied by the data generated for testing and designing multichannel filters.

1. INTRODUCTION

The ambient seismic noise is modeled as a multi-channel Gaussian Markov process. Some arguments for this are based on the following observations:

(1) Sharply tuned spectral peaks at .2 and 2 cps remain fixed for long periods of time.

(2) Principal spectral peaks and the spectrum between the peaks are realized as stationary Gaussian processes on time scales of more than several hours, although some small intermediate frequency peaks are not positive to tests for Gaussian stationarity.

(3) On longer time scales, the power spectrum may be observed to change markedly, but the coherency between channels is comparatively constant.

The synthesized noise combines linearly additive independent stationary Gaussian processes. Two mixed processes are taken to charactize the .2 cps and 2 cps noise peaks. These are modeled with two sharply tuned linear filters. A spatial correlation will result from a Markov chain, in the form

$$\left\{N_{k,t}\right\} = \sum_{1}^{N} a_j \left\{R_{kj,t}\right\} \otimes L_{t-\tau}$$

where $\left\{R_{kj,t}\right\}$ are orthogonal white random functions associated with each channel and $L_{t-\tau}$ is the fixed linear filter response of a noise process. Time independent constants determines a process which is space-linear and which is space-stationary if $R_{kj} = R_{k-j}$.

Although Markov models may be physically interesting as reflecting underlying differential equations of the system, the purpose here is to show that simple models can qualitatively account for many of the observed properties of the noise. Such models are useful for efficiently generating standard test data on a digital computer for checkout of vertical array processors and for preliminary evaluation and ranking of seismic signal processors.

2. MODELING THE LINEAR FILTER RESPONSE AT THE SURFACE OF THE EARTH

Ambient additive noise is represented by white Gaussian inputs to linear filters; the output of the filters are added.

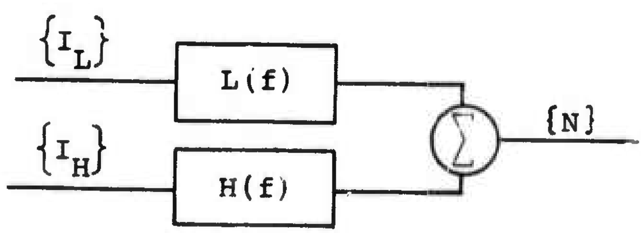


Figure 1

$\{I_L\}$ and $\{I_H\}$ represent independent realizations of white noise from a Gaussian population. L(f) and H(f) are the low and high frequency tuned filters. For example, the filters are associated with the .2 cps microseism peak and the 2 cps noise peaks, respectively. Each tuned filter L(f) and H(f) is modeled by a series of damped oscillators. The frequency response of a tuned filter is determined by selecting appropriate values for the peak frequency f_o and Q (related to bandwidth as $f_o/\Delta f$). In the following equation we represent the spectral realization of noise from such a system.

$$\{N(f)\} = \{C_L(f)\} \left[\frac{1}{1 + \frac{Q_L(f_L^2 - f^2)}{iff_L}}\right]$$

$$+ G\{C_H(f)\} \left[\frac{1}{1 + \frac{Q_H(f_L^2 - f^2)}{iff_L}}\right] = \{L_f\} + \{H_f\} \tag{1}$$

The curley brackets represent the realization of a state from a random population. The samples are statistically independent.

$$E\left(\{C_L(f)\}\ \{\overline{C_H(f)}\}\right) = 0$$

The input random functions are white, with variance or power, P_o.

$$E\left(\{C_L(f)\}\ \{\overline{C_L(f)}\}\right) = E\left(\{C_H(f)\}\ \{\overline{C_H(f)}\}\right) = P_o.$$

The power spectral density corresponding to this noise model has eight unknown parameters adjustable to approximate noise with two principal spectral peaks.

$$E\left(\{N(f)\}\ \{\overline{N(f)}\}\right) = P_o \left[\frac{1}{1 + \left(\frac{Q_L(f_L^2 - f^2)}{ff_L}\right)^2}\right]^{N_L}$$

$$+ GP_o \left[\frac{1}{1 + \left(\frac{Q_H(f_L^2 - f^2)}{ff_L}\right)^2}\right]^{N_H} \tag{2}$$

Writing difference equations for the system, we describe a linear noise prediction filter with constant coefficients. In practice, time realizations of white noise will be based on pseudo random numbers. Specifying the parameters Q, f_o, and N, a recursive filter subroutine RECFIL3 computes the approximate difference equation and filters the white random input.

The low and high frequency component of the power spectral density are given separately.

$$\lambda^2_{L,f} = \left[\frac{1}{1 + \left(\frac{Q_L\,(f_L^2 - f^2)}{f_L f}\right)^2}\right]^{N_L}$$

$$\lambda^2_{H,f} = \left[\frac{1}{1 + \left(\frac{Q_H\,(f_L^2 - f^2)}{f_L f}\right)^2}\right]^{N_H} \tag{2a}$$

3. <u>SPATIAL MARKOV PROCESS</u>

The operator relating noise realizations obtained at different points in space is taken as a separable product of the random temporal functions. The power spectrum of the noise is based on a linear combination of the low and high frequency processes which are treated as statistically independent. The spatial covariance or covariance between channels will then be an interpolation between the Markov models characterizing the two processes. The model which relates the two processes with constant coefficients is too simple to account for all the details of the highly complex dispersive filter response of the earth. However, without backing into a wave mode representation which require boundary values of a known layered earth and a complex matrix describing the mode excitation; the constants relating the two processes on a channel can be extended to complex dispersive structures by convolution operators. For efficient generation of large quanties of test data, it is sufficient only to model the system with constants, or with simple operators (e.g., through time shifting, difference operators, etc.). Coefficients selected should naturally lead to a noise covariance structure approximating the noise in the signal band.

Vertical array sensors sample time fluctuations at a set of discrete points designated as channels. The random time function on channel i corresponding to either the low or high frequency process is described as a state of channel i. There are N channels denumerated from that closest to the surface in order of increasing distance from the surface. The system is limited to 2N orthogonal states, each associated uniquely with a process on each channel. By means of a first order Markov chain applied to each process the noise on channel i+1 is related to noise on channel i. Starting with the first channel, we restate (1) with more compact notation.

$$\left\{N_{1,f}\right\} = c_1 \, (1-a_1\bar{a}_1)^{\frac{1}{2}} \left\{L_{1,f}\right\} + d_1 (1-b_1\bar{b}_1)^{\frac{1}{2}} \left\{H_{1,f}\right\} \tag{3}$$

i^{th}

The noise on the other channels are related through first order linear spatial difference operators for which the power spectral density of the noise is invariant.

$$\{N_{k,f}\} = c_k\left[a_k\{L_{k-1,f}\} + (1-a_k\bar{a}_k)^{\frac{1}{2}}\{L_{k,f}\}\right] + d_k\left[b_k\{H_{k-1,f}\} + (1-b_k\bar{b}_k)^{\frac{1}{2}}\{H_{k,f}\}\right] \tag{4}$$

where

$$a_1 = b_1 = 0 \qquad E\left(\{L_{i,f}\}\overline{\{H_{j,f}\}}\right) = 0$$

$$E\left(\{L_{i,f}\}\overline{\{L_{j,f}\}}\right) = \lambda^2_{L,f}\delta_{i-j}, \quad E\left(\{H_{i,f}\}\overline{\{H_{j,f}\}}\right) = \lambda^2_{H,f}\delta_{i-j}$$

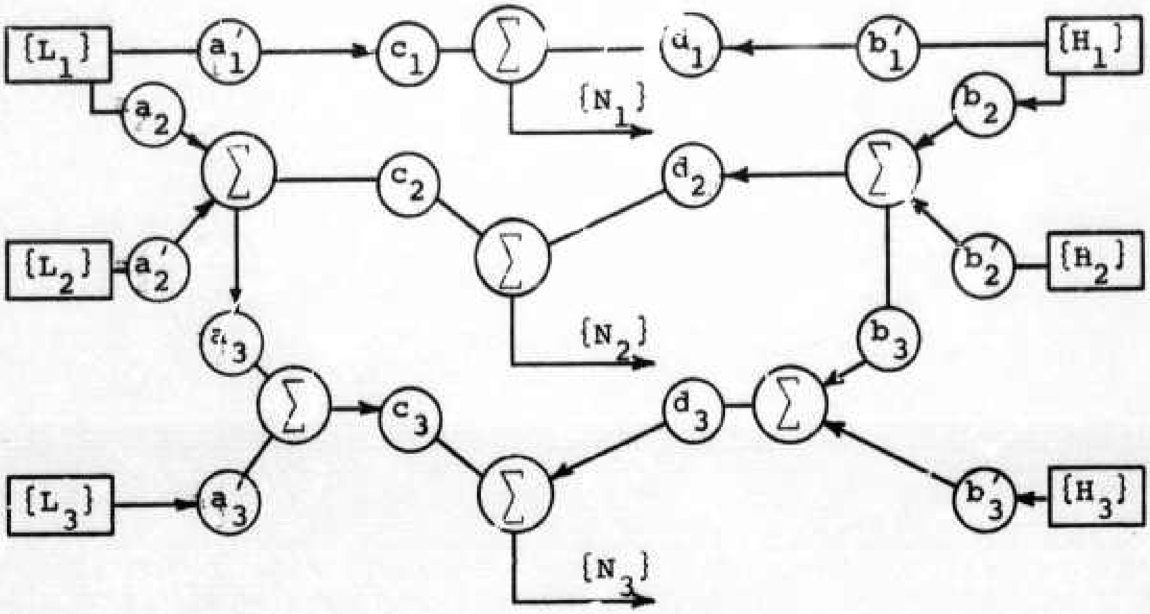
Applying (4) to (3), we generate noise on channel i.

$$\{N_{i,f}\} = c_i\sum_{1}^{i}\left(\prod_{k+1}^{i} a_j\right)(1-a_k\bar{a}_k)^{\frac{1}{2}}\{L_{k,f}\} + d_i\sum_{1}^{i}\left(\prod_{k+1}^{i} b_j\right)(1-b_k\bar{b}_k)^{\frac{1}{2}}\{H_{k,f}\} \tag{5}$$

The solution for (3) and (4) given by (5) describes a general linear first order Markov chain with the initial realization taken from the channel closest to the earth's surface. A schematic of the system is shown on Figure 2, for three channels. As a special case, the parameter a_i and b_i ($i > 1$) are constant coefficients, the noise power spectrum is the same on each channel.

$$\{N_{i,f}\} = c_1\left[a^{i-1}\{L_{1,f}\} + \sum_{2}^{i} a^{i-k}\{L_{k,f}\}\,(1-a\bar{a})^{1/2}\right]$$

$$+\, d_1\left[b^{i-1}\{H_{1,f}\} + \sum_{2}^{i} b^{i-k}\{H_{k,f}\}\,(1-b\bar{b})^{1/2}\right] \tag{6}$$



$a'_m = (1-a_m\bar{a}_m)^{1/2}$ $\qquad$ $b'_m = (1-b_m\bar{b}_m)^{1/2}$

Figure 2

The scaler gains on each channel given by c_i and d_i may be generalized by associating a convolution operator of one or more time delays. This synthesizes a frequency dependent relationship between noise on two adjacent channels. This could be of some interest in accurately modeling a complex dispersive system.

4. SPECTRAL COVARIANCE

Samples of seismic noise are obtained by implementing a vertical array of seismometers and storing the noise measurements on magnetic tape. The noise is sampled and digitized at equal time intervals. Suppose we record a sample of N points on channel j. A sequence of such records can be ordered by the index k. For each of the realizations, the spectrum of the noise can be computed from zero frequency to the nyquist frequency 1/2 ΔT, with resolution 1/2T, where ΔT is the fixed sampling time-interval and T is the fixed half time length of the noise sample. For realization k on channel j, this is represented as ${}^{k}\left\{N_{j,f}\right\}$, a sequence of complex random numbers.

The noise associated with the system is assumed zero mean, stationary and Gaussian. Thus the statistics are completely characterized by the variance covariance matrix. This is usually given by the array of auto-correlation functions and cross-correlation functions which in principal are computed with desired precision from a sample of multi-channel noise of sufficient time-length. Each function in the array of correlation functions can be spectral analyzed. The resulting array of spectrums is called the power spectral density of the system. According to the ergodic hypothesis, the power spectral density functions can also be obtained by dividing the multichannel noise sample into segments of fixed half time-length T, as ${}^{k}\left\{N_{j,t}\right\}_{-T}^{+T}$, where 1/2T is the desired resolution in cycles per second. By Fourier analysis of each record we obtain <u>sample</u> spectral realizations ${}^{k}\left\{N_{j,f}\right\}$ (this can now be done efficiently with the digital computer). The array of functions representing the power spectral density of the system is given by averaging the product of sample spectral realizations.

$$P_{rs,f} = E\left(\{N_{r,f}\} \{\overline{N_{s,f}}\} \right) = \lim_{N\to\infty} \left[\frac{1}{N} \sum_{1}^{N} {}^{k}\{N_{r,f}\} \, {}^{k}\{\overline{N_{s,f}}\} \right] \qquad (7)$$

In practice, N is finite and we obtain a sample power spectral density.

$$S_{rs,f} = \frac{1}{N} \sum_{1}^{N} {}^{k}\{N_{r,f}\} \, {}^{k}\{\overline{N_{s,f}}\} \qquad \begin{array}{l} r = 1,2,\ldots\ldots\ldots\ldots,k \text{ channels} \\ s = 1,2,\ldots\ldots\ldots\ldots,k \text{ channels} \end{array} \qquad (8)$$

Since $S_{rs,f}$ is Hermitian, K real and (K-1)! complex numbers need be computed for each frequency.

Errors in $S_{rs,f}$ computed from real seismic data lead to considerable difficulty. Even if the sample realizations are from a Gaussian population it is difficult to derive its probability distribution or confidence limits. An additional problem is evaluation of the degree to which the sample is representative of a stationary Gaussian population. More specifically, how should bias and scatter from non-stationary or higher moment components be accounted for? These problems of error analysis and stability of estimates are compounded by using sample covariance estimates to derive least squares operators. These stem from the possible magnification of sampling errors in inverting the sample power spectral density matrix.

For preliminary testing and comparison of filters, a statistical model such as that described in the previous section offers many advantages. Test data can be standardized and used for performance evaluation and comparison of various processors. It is flexible enough for designing models which reasonably approximate the noise on any physically-realized vertical array. The generation of test data by the computer is highly efficient.

Most importantly, the spectral covariance of the noise defined by (7) can be theoretically derived, is exact, and is subject only to roundoff errors rather than the much larger uncertainties of sample variance calculations. Since the noise generated for testing is Gaussian and stationary, derived operators can be validly used outside of the fitting interval. For the Gaussian Markov model given by (5), the theoretical power spectral matrix is derived in Appendix 1, with the results given by (9).

$$P_{rs,f} = c_r\bar{c}_s \left(\overline{\prod_{r+1}^{s} a_j} \right) \lambda^2_{L,f} + d_r\bar{d}_s \left(\overline{\prod_{r+1}^{s} b_j} \right) \lambda^2_{H,f} \tag{9}$$

It can easily be shown that $P_{rs,f}$ is Hermitian, $P_{sr,f} = \bar{P}_{rs,f}$. The power spectral density matrix can be computed from (9). The terms $\pi\,\bar{a}_j$ and $\pi\,\bar{b}_j$ yield information on the net stepout and attenuation in the cross-power terms due to complex coefficients in the Markov chain. The terms $c_r\,\bar{c}_s$ and $d_r\,\bar{d}_s$ also can yield net stepouts. Also, for a highly complex dispersive medium $c_r\,\bar{c}_s$ and $d_r\bar{d}_s$ are frequency dependent. No unique explanation can be given for stepouts in the noise, they may occur due to a number of effects such as body wave conversion, leaky modes, complex coupling between propagation modes, or complex dissipating mechanisms. It should be clear, however, that a simple Markov process can be used to rapidly generate noise samples which reasonably approximate the average noise power spectral density matrix in the signal band.

5. VERTICAL ARRAY TRANSIENT SIGNAL MODEL

In line with our objective of efficiently generating test data, the signal is modeled from a simple pulse formula that looks like a teleseismic wavelet. No attempt is made to physically relate the synthetic signal to such phenomena as absorption, displacement potential, etc. since we are only interested in testing filters. In testing filters for signal distortion, it is useful to generate signals ranging from a high frequency near 2 cps and low frequency near 1 cps. Since this reasonably covers a range of expected signal wave forms, signal distortion evaluations on a suite of such signals provides a robust test for signal distortion of filters. For example, if a signal distortion test passes all of the test signals, then the distortion can probably be tolerated for any expected teleseismic signal.

The function used to generate the seismic wavelet has a single spectral peak at frequency f_o and bandwidth controlled by the parameter m. Larger values of m gives smaller bandwidths. Using m of the order of 100, the signal looks somewhat like a Ricker wavelet.

$$W(t') = \left[4\pi^2 f^2 t'^2 (m/2-1) + 4\pi f t' (1-m)(m/2-1)^{\frac{1}{2}} + (m-1)(m-2)\right]$$

$$t'^{(m-3)} \exp\left[-2\pi f t' (m/2-1)^{\frac{1}{2}}\right] \quad t'>0 \tag{10}$$

where

$t' = t-t_o$, the reduced time

and $W(t) = 0 \quad t'<0$

For completeness, the wavelet generator includes the pP, specified by the echo-time and the apparent surface reflection coefficient at the source. Further, the vertical array p-ghost is accounted for by specifying the echo-time and surface reflection coefficient at the receiver.

6. RESULTS

As an example, noise on a vertical array is simulated using (6). The linear filter describing the noise of each channel is taken from (1) with $N_L = 2$, $N_H = 1$, $Q_L = 3$, $Q_H = 6$, $f_L = .2$cps, $f_H = 2.$ cps and $G = .012$. This provides a reasonable approximation of the power spectral density of a sample of noise from Apache, Oklahoma. The spectral power is shown on Figure 1. The spatial Markov chain is obtained from (3) and (4). Comparing (3) and (2).

$$E\left(\{N_1(f)\}\overline{\{N(f)\}}\right) = P_o\lambda^2_{L,f} + GP_o\lambda^2_{H,f} = c_1\bar{c}_1\ (1-a_1\bar{a}_1)$$

$$\lambda^2_{L,f} + d_1\bar{d}_1\ (1-b_1\bar{b}_1)\ \lambda^2_{H,f}$$

From comparison with the power spectrum of a noise sample, we take

$$a_1 = b_1 = 0 \quad c_1\bar{c}_1 = P_o = 25. \quad d_1\bar{d}_1 = GP_o = 1.$$

$$a_i = .9 \quad b_i = .6_{(i > 1)} \quad c_1 = c_i = 5. \quad d_1 = d_i = 1.$$

For the constants describing the Markov chain, we take $b_i = .5$ reflecting small spatial correlation for the 2 cps components; and $a_i = .9$ which determines a .9 ordinary coherency between adjacent channels for the .2 cps noise component. Although this is a simple space structure, it provides the following characteristics; the coherency decreases with increasing frequency starting near .9 at .2 cps and reaching a low value at 2 cps; and the coherency between channels falls off proportional to distance. Further, as control in utilizing the synthetic data for filtering experiments, the power spectral density of the noise is made identical on each channel. Figure 1 shows the power spectrum of the noise. Figure 2 shows a sample realization of synthetic noise for a six element vertical array. Figure 3 shows a plot of the ordinary coherencies against frequency. Figure 4 shows a matrix plot of the power spectral density, which is real for this particular example. Figure 5 shows six different signal models taken from (10).

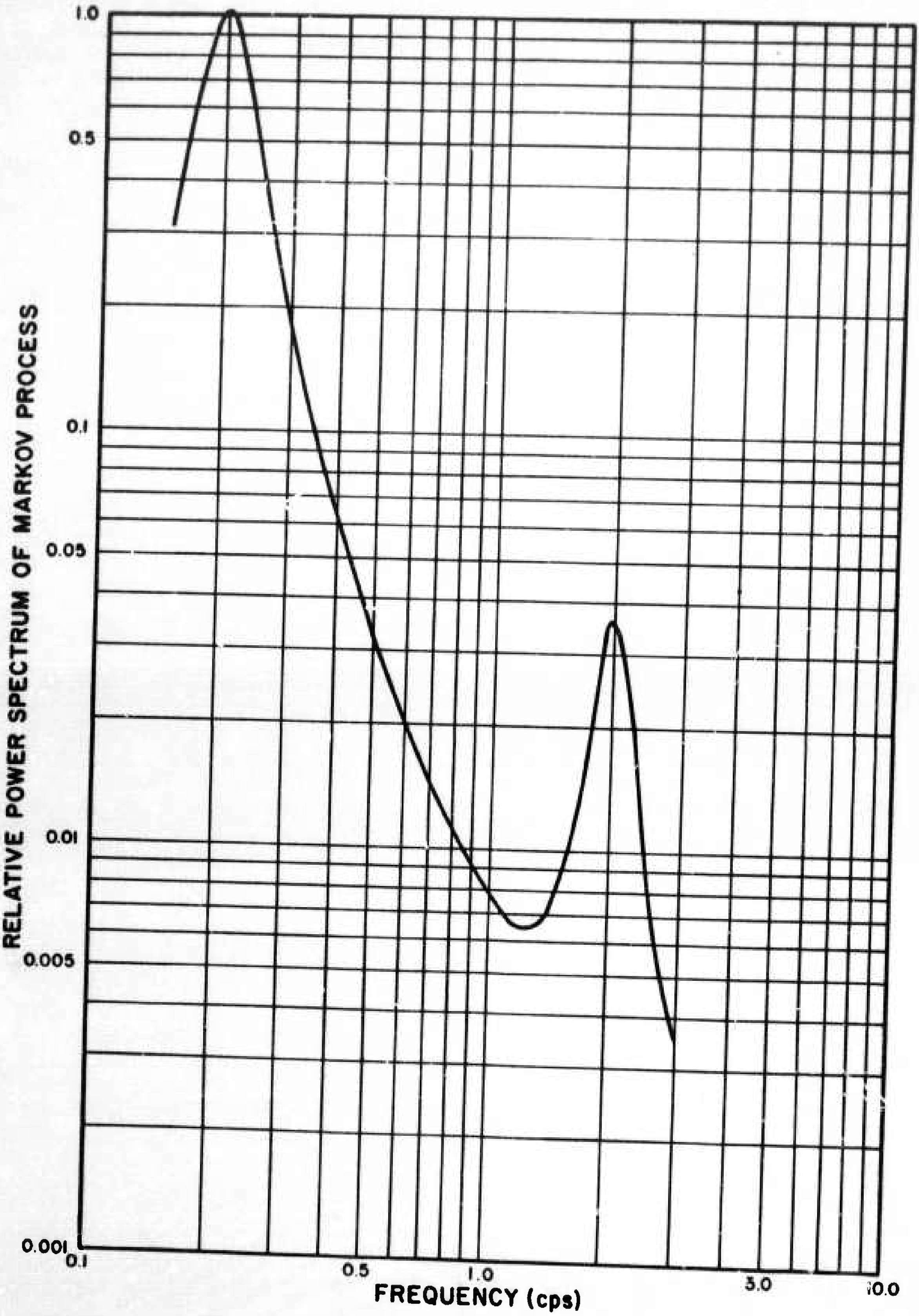


Figure 1. Power Spectrum of Noise on Each Channel

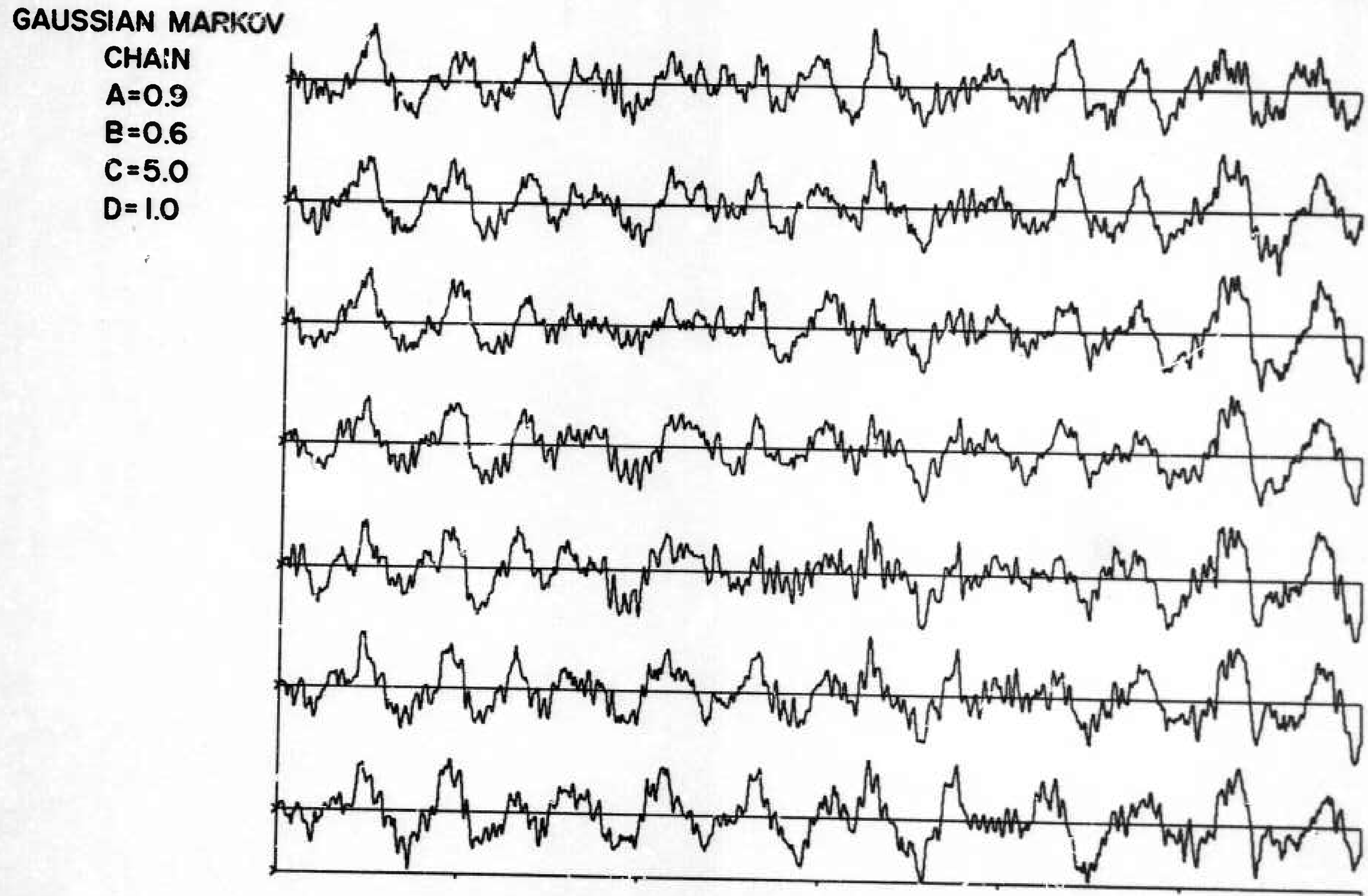


Figure 2. Vertical Array Noise

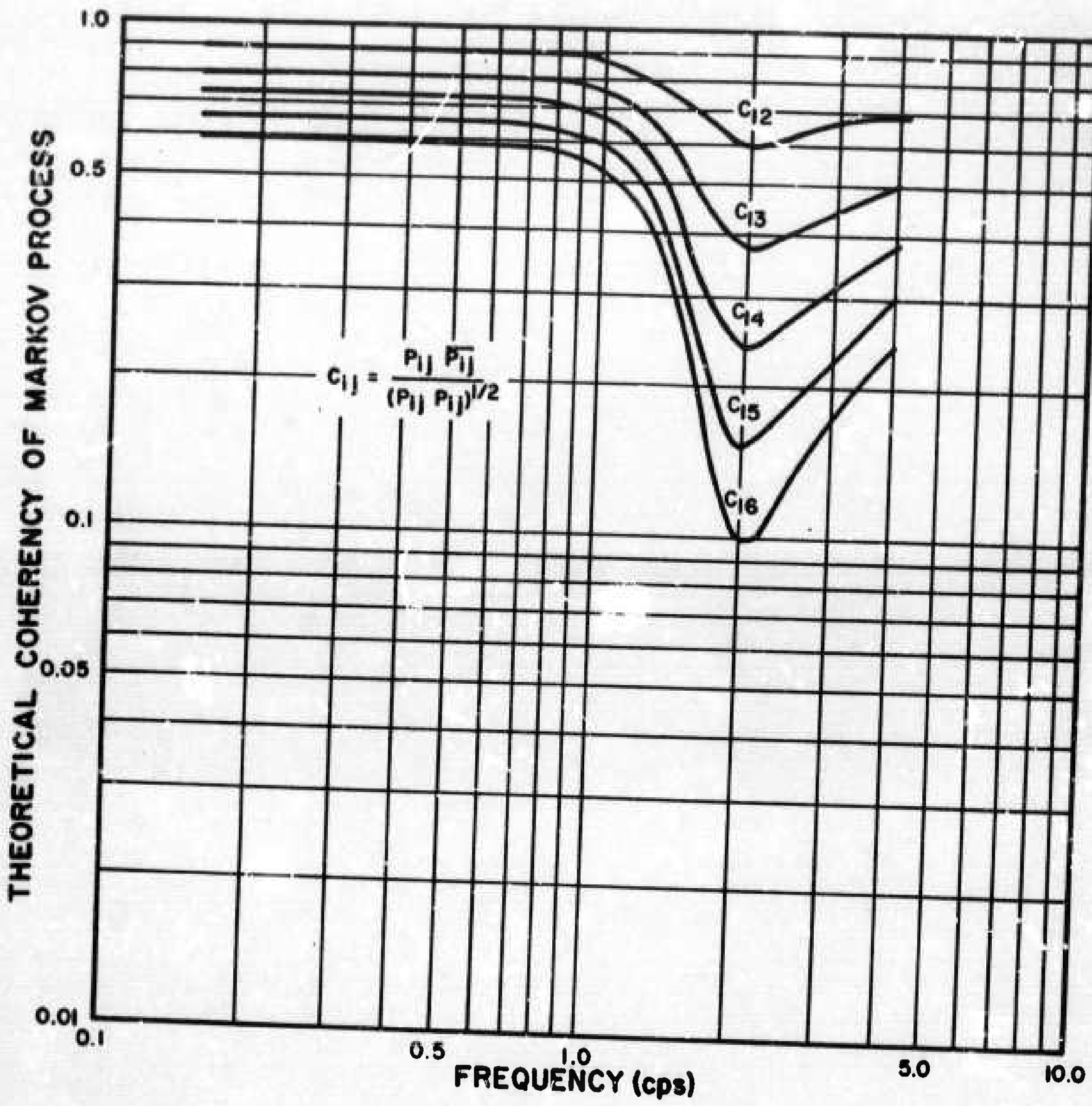


Figure 3. C_{ij} = Coherency Between i^{th} and j^{th} channel

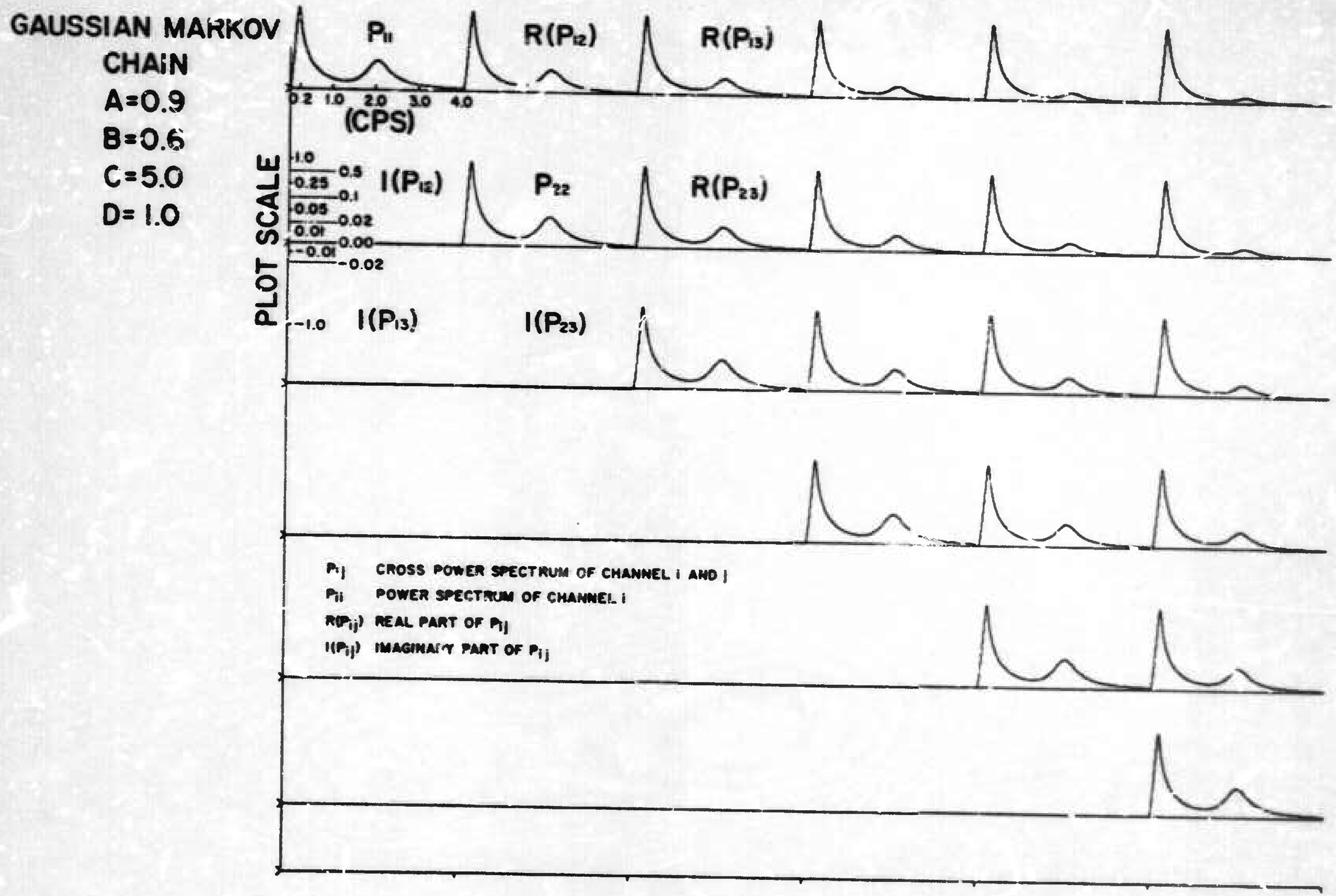


Figure 4. Power Spectral Matrix Plot

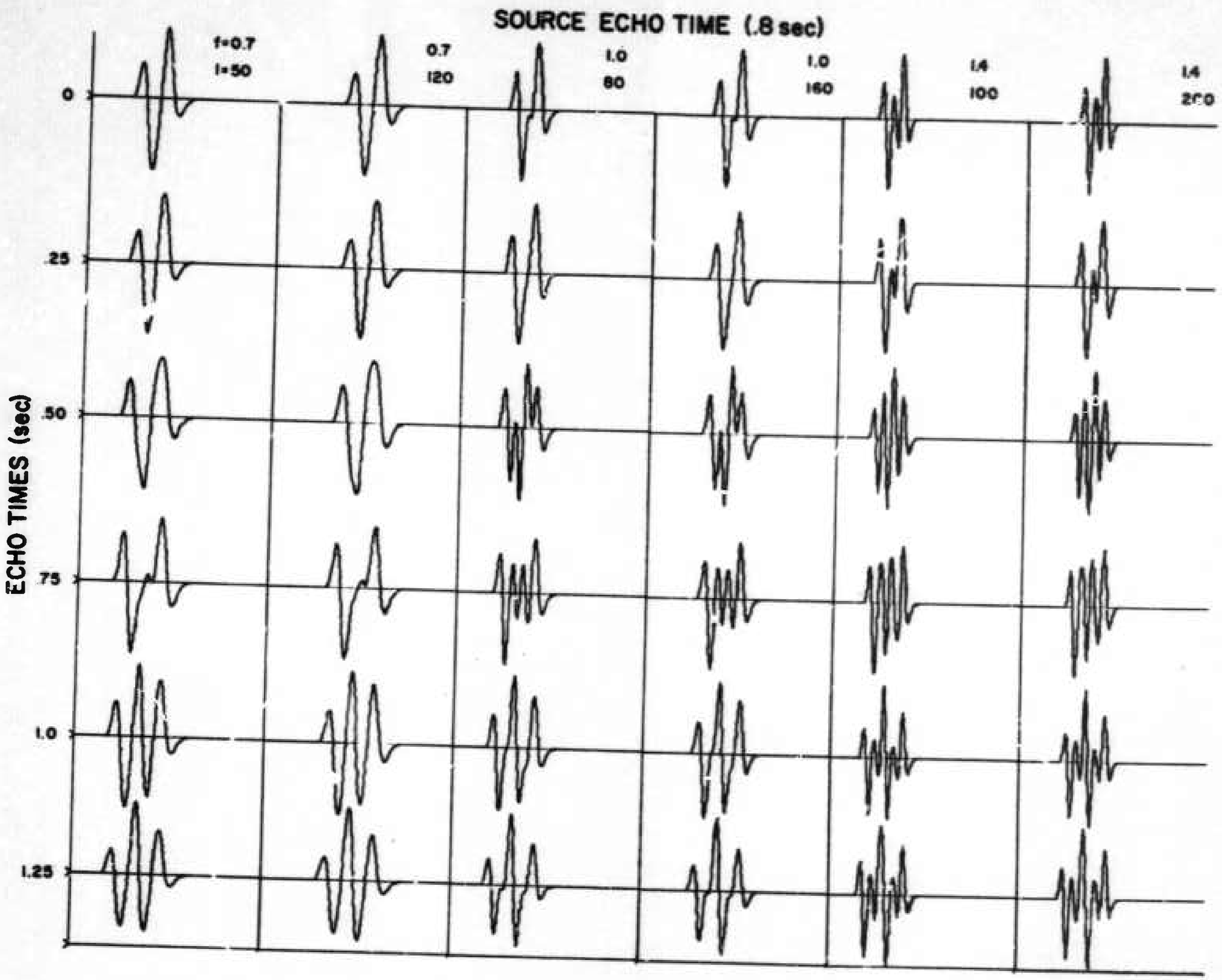


Figure 5. Vertical Array Signals

APPENDIX 1

Derivation of Power Spectral Density Matrix

$$P_{rs,f} = c_r \bar{c}_s \sum_1^r \sum_1^s \left(\prod_{k+1}^r a_j \right) \left(\prod_{m+1}^s a_j \right) (1-a_k a_k)^{\frac{1}{2}} (1-a_m a_m)^{\frac{1}{2}}$$

$$+ d_r \bar{d}_s \sum_1^r \sum_1^s \left(\prod_{k+1}^r b_j \right) \left(\prod_{m+1}^s b_j \right) (1-b_k b_k)^{\frac{1}{2}} (1-b_m b_m)^{\frac{1}{2}}$$

The noise realizations are taken orthogonal as from (2a) and (2b).

$$P_{rs,f} = c_r \bar{c}_s \sum_1^r \left(\prod_{k+1}^r a_j \bar{a}_j \right) \left(\prod_{r+1}^s \bar{a}_j \right) (1-a_k \bar{a}_k) \; \lambda_{L,f}^2$$

$$+ d_r d_s \sum_1^r \left(\prod_{k+1}^r b_j \bar{b}_j \right) \left(\prod_{r+1}^s \bar{b}_j \right) (1-b_k \bar{b}_k) \; \lambda_{H,f}^2$$

$$= c_r \bar{c}_s \left(\prod_{r+1}^s \bar{a}_j \right) \left[\sum_1^r \left(\prod_{k+1}^r a_j \bar{a}_j \right) - \sum_1^r \left(\prod_k^r a_j \bar{a}_j \right) \right]$$

$$\lambda_{L,f}^2$$

$$+ d_r d_s \left(\prod_{r+1}^s \bar{b}_j \right) \left[\sum_1^r \left(\prod_{k+1}^r b_j \bar{b}_j \right) + \sum_1^r \left(\prod_k^r b_j \bar{b}_j \right) \right]$$

$$\lambda_{H,f}^2$$

$$= c_r \bar{c}_s \left(\prod_{r+1}^s \bar{a}_j \right) \left[1 - \prod_1^r a_j \bar{a}_j \right] \; \lambda_{L,f}^2$$

$$+ d_r\bar{d}_s \left(\prod_{r+1}^{s} \bar{b}_j \right) \left[1 - \prod_{1}^{r} b_j \bar{b}_j \right] \lambda^2_{H,f} \quad r < s$$

without loss in generality we may set $a_1 = b_1 = 0$ so that $\prod_{r+1}^{s} a_j\bar{a}_j = \prod_{r+1}^{s} b_j\bar{b}_j = 0$ for $r < s$.

$$P_{rs,f} = c_r\bar{c}_s \overline{\left(\prod_{r+1}^{s} a_j \right)} \lambda^2_{L,f} + d_r\bar{d}_s \overline{\left(\prod_{r+1}^{s} b_j \right)}$$

$$\lambda^2_{H,f} \quad r < s \text{ and } P_{sr,f} = \bar{P}_{rs,f}.$$

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13. ABSTRACT

The ambient seismic noise is modeled by a single Gaussian population from which independent realizations or states are taken as input to tuned filters with spectral peaks matched to those observed in noise samples, for example at .2 cps and 2. cps. For each spectral noise peak, the realization on channel i + 1 is equal to a constant times the realization on channel i plus another constant times a new realization on channel i + 1. The constants defining the Markov process can be used to theoretically derive the associated power spectral matrix of the noise model. The model can be extended to dispersive systems by using a set of constants and time lags to relate the noise on channel i to that on channel i + 1.

A vertical array signal model is also given. The purpose is to efficiently generate noise and/or signals at prescribed S/N ratios. The noise covariance structure is close to that observed naturally and is known exactly for the noise model realizations. Thus the spectral covariance of the noise is given exactly subject only to roundoff error, and conditions of stationarity and equilibrium are satisfied by the data generated for testing and designing multichannel filters.

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14. KEY WORDS	LINK A ROLE	LINK A WT	LINK B ROLE	LINK B WT	LINK C ROLE	LINK C WT
Vertical Array						
Multichannel noise						
Gauss-Markov process						
Stationary noise						
Linear Noise model						
Principal peaks in noise spectrum						
Superposition of damped oscillators						
Spatial Markov process						
Spectral Covariance						
Coherency						
Power spectrum						
Signal Model						

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